CHAPTER 9

Cost-Volume-Profit Analysis

Introduction

- Cost-Volume-Profit (CVP) Analysis is a method for analyzing how operating decisions and marketing decisions affect profit.
- The decisions are based on an understanding of the relationship between variable costs, fixed costs, unit selling price, and the output level.
- CVP analysis has many applications:
  - Setting prices for products and services.
  - Introducing a new product or service.
  - Replacing a piece of equipment.
  - Determining the breakeven point.
  - Deciding whether to make or buy a given product or service.
  - Determining the best product mix.
  - Performing strategic what-if analyses.

Introduction

- CVP analysis is based on an explicit model of the relationships among the three factors:
  - costs, sales, and profits—and how they change in a predictable way as the volume of activity changes.
- The CVP model is

  Operating profit = Sales - Total costs
- Since we will be looking at how costs and sales vary with volume.
- It is important to distinguish variable and fixed costs, and to show the above equation in the equivalent form below:

Introduction

$$\text{Sales} = \text{Fixed costs} + \text{Variable costs} + \text{Operating profit}$$

Now, replacing sales with the number of units sold times price, and replacing variable cost with unit variable cost times the number of units sold, the CVP model is

$$\begin{aligned}\text{Units sold} \times \text{Price} = {} & \text{Fixed cost} \\ & + \text{Units sold} \times \text{Unit variable cost} \\ & + \text{Operating profit}\end{aligned}$$

For convenient use, the model is commonly shown in a symbolic form:[1]

Q = units sold

p = unit selling price

F = total fixed cost

v = unit variable cost

N = operating profit

$$p \times Q = F + (v \times Q) + N$$

Contribution Margin and Contribution Income Statement

- Effective use of the CVP model requires an understanding of three additional concepts:
  - The contribution margin
  - The contribution margin ratio, and
  - The contribution income statement
- The **unit contribution margin** is the difference between unit sales price and unit variable cost:

$$p - v = \text{Unit contribution margin}$$

- The unit contribution margin measures the increase in operating profit for a unit increase in sales.
- If sales are expected to increase by 100 units, profits should increase by 100 times the contribution margin.

Contribution Margin and Contribution Income Statement

- The **total contribution margin** is the unit contribution margin multiplied by the number of units sold.

- For example, assume that Household Furnishings, Inc. (HFI), a manufacturer of home furnishings, is interested in developing a new product, a wooden TV table, that would be priced at $75 and would have variable costs of $35 per unit. Assume also that the new product will have no effect on sales of existing products. The investment would require new fixed costs of $5,000 per month ($60,000/yr). HFI expects sales of 2,400 units in the first year and 2,600 units in the second year. The data for HFI are summarized in Exhibit 9.1 .

Contribution Margin and Contribution Income Statement

- A measure of the profit contribution per sales dollar is the **contribution margin ratio,** which is the ratio of the unit contribution margin to unit sales price (p - v)/ p.
- A useful way to show the information developed in CVP analysis is to use the contribution income statement.
- The **contribution income statement** puts the focus on cost behavior.
- It separates fixed costs and variable costs.

Contribution Margin and Contribution Income Statement

	Per Unit	2010	2011
Fixed cost		$60,000	$60,000
Revenue	$75		
Variable cost	35		
Planned production		2,400 units	2,600 units
Planned sales		2,400 units	2,600 units

EXHIBIT 9.2 Contribution Income Statements for HFI's Proposed TV Table

	2010		2011			
	Amount	Percent	Amount	Percent	Change	Notes
Sales	$180,000	100.00%	$195,000	100.00%	$15,000	
Variable costs	84,000	46.67	91,000	46.67	7,000	
Total contribution margin	$ 96,000	53.33%	$104,000	53.33%	$ 8,000	53.33% is the contribution margin ratio
Fixed costs	60,000		60,000		0	
Operating profit	$ 36,000		$ 44,000		$ 8,000	$8,000 = 0.5333 × $15,000

Strategic Role of CVP Analysis

1. What is the expected level of profit at a given sales volume?
2. What additional amount of sales is needed to achieve a desired level of profit?
3. What will be the effect on profit of a given increase in sales?
4. What is the required funding level for a governmental agency, given desired service levels?
5. Is the forecast for sales consistent with forecasted profits?
6. What additional profit would be obtained from a given percentage reduction in unit variable costs?
7. What increase in sales is needed to make up a given decrease in price to maintain the present profit level?
8. What sales level is needed to cover all costs in a sales region or product line?
9. What is the required amount of increase in sales to meet the additional fixed charges from a proposed plant expansion?

CVP Analysis for Breakeven Planning

- The starting point in many business plans is to determine the **breakeven point,** the point at which revenues equal total costs and profit is zero.
- The CVP model is solved by inserting known values for unit variable cost (*v*), price (*p*), and total fixed cost (*F*), setting desired before tax profit (*N*) equal to zero, and then solving for *Q.*
- Here we assume that fixed costs cannot be changed over the planning period.
- We can solve for *Q* in two ways:
  - The equation method and
  - The contribution margin method.
- Each method can determine the breakeven point in units sold or sales dollars.

Equation Method: For Breakeven in Units

The equation method uses the CVP model directly. For example, using information from Exhibit 9.2, the equation for the analysis of HFI's sale of TV tables is

$$\text{Sales} = \text{Fixed cost} + \text{Total variable cost} + \text{Operating profit}$$

$$p \times Q = F + (v \times Q) + N$$

$$\$75 \times Q = \$5{,}000 + (\$35 \times Q)$$

Solving for Q and assuming $N = 0$ the breakeven point is $Q = 125$ TV tables per month (1,500 units per year).

$$(\$75 - \$35) \times Q = \$5{,}000$$

$$Q = \$5{,}000/(\$75 - \$35)$$

$$Q = \$5{,}000/\$40 = 125 \text{ units per month}$$

Equation Method: For Breakeven in Units

- The contribution to profit per TV table is measured directly by the unit contribution margin, *p - v,* which is $40 per table.
- So, since at sales of 125 units the profit is zero, at 126 units the profit is $40, at 127 units the profit is 2 $40 $80, and so on.
- Using the unit contribution margin gives us a quick way to determine the change in profit for a change in sales units. At the 128-unit level, profit is

Sales: 128 units at $75/unit	$9,600
Less:	
Variable costs: 128 at $35/unit	4,480
Contribution margin	$5,120
Fixed costs	5,000
Operating profit	$ 120

Equation Method: For Breakeven in Dollars

- Sometimes units sold, unit variable cost, and sales price are not known, or it is impractical to determine them.
- For example, suppose that a firm has many products and is interested in finding the overall breakeven level for all products taken together. It is not practical to find the breakeven in units for each product, but it is possible to find the breakeven in sales dollars for all products.
- We use the equation method in a revised form, where Y is the breakeven point in *sales dollars.*
- This model is equivalent to the model used for breakeven in units, except that Q is replaced by Y / p (i.e., sales in dollars divided by price = quantity; $Y / p = Q$), as follows:

$$\text{Sales} = \text{Fixed Cost} + \text{Total Variable Cost} + \text{Profit}$$

$$p \times Q = F + (v \times Q) + N$$

$$p \times (Y/p) = F + [v \times (Y/p)] + N$$

$$Y = F + [(v/p) \times Y] + N$$

Continuing with the HFI data in Exhibit 9.2, assume that because there are many different products, we know only total annual variable cost ($84,000), total annual sales ($180,000) and total monthly fixed costs ($5,000). We can obtain the ratio, $v/p = 0.4667$ ($84,000/$180,000), and solve for breakeven in dollars:

$$Y = (0.4667 \times Y) + \$5,000$$

$$Y = \$9,375 \text{ per month (125 units at \$75 each)}$$

Contribution Margin Method

- A convenient method for calculating the breakeven point is to use the equation in its equivalent algebraic form (derived by solving the model for *Q* and assuming at breakeven *N* = profit = 0):

$$Q = \text{Fixed costs/Unit contribution margin}$$

$$= \frac{F}{p - v}$$

The contribution margin method (so-called because the contribution margin is the denominator of the ratio) produces the same result as the equation method:

$$Q = (\$5,000)/(\$75 - \$35) = 125 \text{ units per month}$$

The contribution margin method can also be used to obtain breakeven in dollars, using the contribution margin ratio (by solving $Y = F + [(v/p) \times Y]$)

$$Y = \frac{F}{(p - v)/p}$$

where:

$$(p - v)/p = \text{the contribution margin ratio}$$

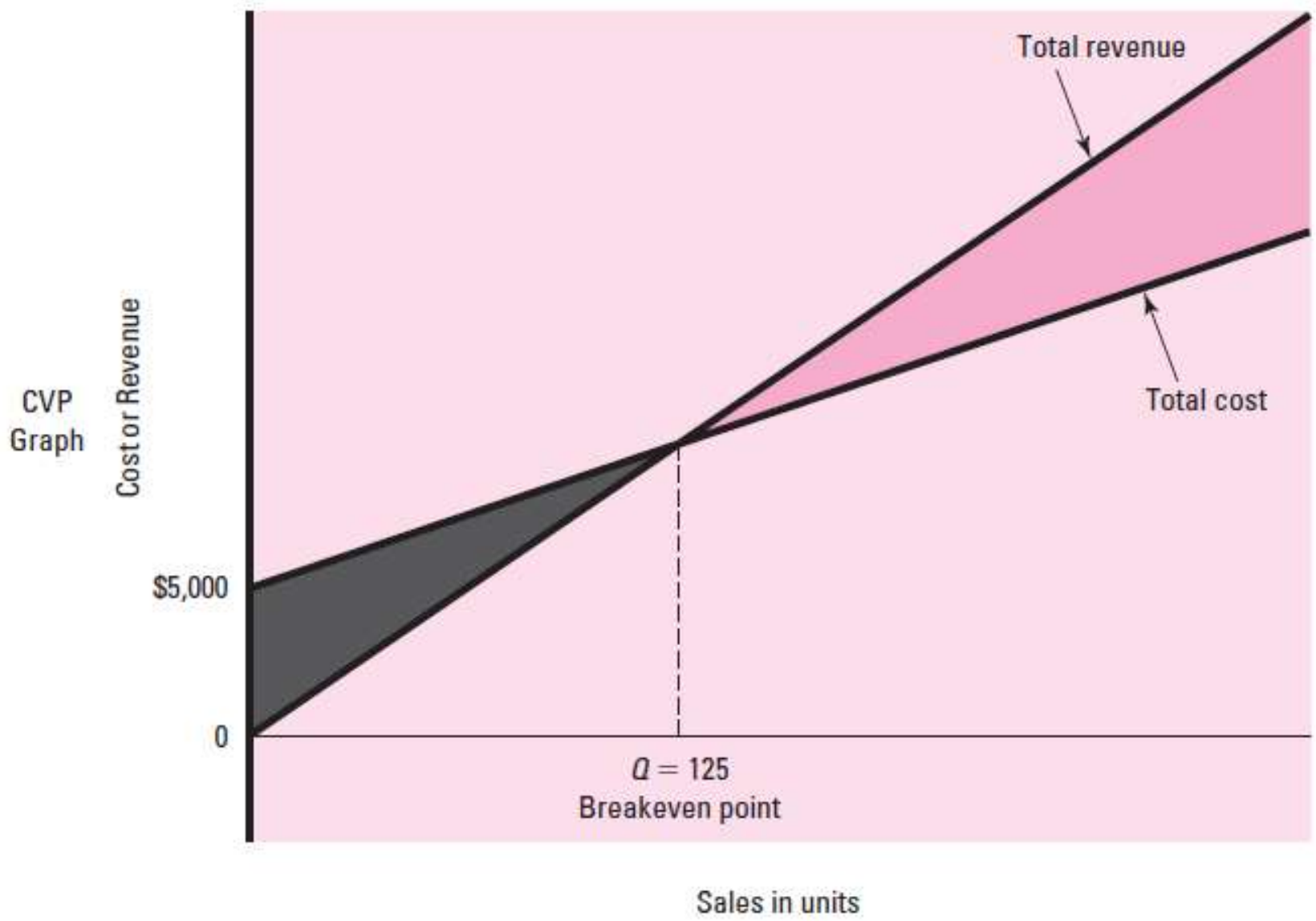
For the HFI example, the contribution margin ratio is 0.5333 (the same as one minus the v/p ratio of .4667, as calculated above).

$$Y = \$5{,}000/0.5333 = \$9{,}375 \text{ per month}$$

- Some find the equation method easier to use, and others prefer to use the contribution margin method. Both methods produce the same results.

CVP Graph and the Profit-Volume Graph

- Breakeven analysis is illustrated graphically.
- It shows the CVP graph at the top and the profit-volume graph beneath.
- The **CVP graph** illustrates how the levels of revenues and total costs change over different levels of sales volume in units.
- The **profit-volume graph** in the lower portion of the exhibit illustrates how the level of profits changes over different levels of sales.
- The slope of the profit-volume line is the unit contribution margin; therefore, the profit-volume graph can be used to read directly how total contribution margin and profit changes as the sales level changes.


CVP
Graph
Cost or Revenue
Total revenue
Total cost
$5,000
0
Q = 125
Breakeven point
Sales in units


Profit-Volume Graph
Profits
0
–$5,000
Q = 125
Profits
Losses
Sales in units

Summary of Breakeven Methods

The Equation Methods

1. Breakeven in sales units (Q = sales in units)

$$p * Q = (v * Q) + F + N$$

2. Breakeven in sales dollars (Y sales in dollars)

$$Y = [(v/p) * Y] + F + N$$

The Contribution Margin Methods

3. Breakeven in sales units =

$$Q = \frac{F + N}{p - v}$$

4. Breakeven in sales dollars =

$$Y = \frac{F + N}{(p - v)/p}$$

CVP Analysis for Profit Planning

- CVP analysis can be used to determine the level of sales needed to achieve a desired level of profit.
- **Revenue Planning**
  - CVP analysis assists managers in revenue planning to determine the revenue required to achieve a desired profit level.
- For example, if HFI's management wants to know the sales volume necessary to achieve $48,000 in annual profits, we substitute $60,000 for fixed costs and $48,000 for desired profit;

$$Q = \frac{F + N}{p - v} = \frac{\$60,000 + \$48,000}{\$75 - \$35} = 2,700 \text{ units per year}$$

The solution in sales dollars is

$$p \times Q = \$75 \times 2,700 = \$202,500 \text{ per year}$$

Cost Planning

Trade-Offs between Fixed and Variable Costs—Example One (One Machine)

- To continue with the HFI example, assume sales of 2,700 units per year. Management is now considering the purchase of a new machine that will reduce variable costs but also increase fixed costs by $2,250 per month. How much must unit variable costs fall to maintain the current level of profit, assuming that sales volume and all other factors remain the same?

$Q = 2{,}700$ units $\qquad F = \$5{,}000 + \$2{,}250 = \$7{,}250$ per month ($87,000 per year)

$p = \$75$ $\qquad N = \$48{,}000$ per year

$v =$ an unknown (previously \$35)

Now, instead of solving for Q (which is given as 2,700 units), we solve for v, as follows:

$$Q = \frac{F + N}{p - v}$$

$$p - v = \frac{F + N}{Q}$$

$$v = p - \frac{F + N}{Q}$$

$$v = \$75 - [(\$87{,}000 + \$48{,}000)/2{,}700] = \$25$$

Trade-Offs between Fixed and Variable Costs—Example Two (Two Machines)

- Assume as in example one that management is considering the purchase of a new machine and, in this case, has the choice between two machines.
- For example, a particular machine might have a relatively high purchase cost but will provide lower operating costs in comparison to an alternative machine.
- So there is an option to trade off between high fixed costs and low unit variable costs (the high fixed-cost option) versus relatively low fixed costs and relatively higher unit variable costs (the low fixed-cost option).
- As volume increases, the high fixed cost option will be more and more attractive, since it brings a reduction in total variable costs.
- Breakeven analysis can help to find the level of sales (called the *indifference point*), such that having sales greater than this level will favor the high fixed-cost option, and sales less than this level will favor the low fixed-cost option.

Example

- Two assumes HFI can choose between two machines, either of which will complete the same operation with the same quality, but with different fixed and variable costs. Machine A has a fixed cost of $5,000 and a unit variable operating cost of $10, while machine B has a fixed cost of $15,000 and unit variable operating cost of $5. To find the indifference point, where the low fixed-cost (A) and high fixed-cost (B) options have the same total costs, we set the cost equations for each option equal and solve for sales quantity.

$$\begin{aligned} \text{Cost of machine A} &= \text{Cost of machine B} \\ \$5{,}000 + (\$10 \times Q) &= \$15{,}000 + (\$5 \times Q) \\ Q &= \$10{,}000/\$5 = 2{,}000 \text{ units} \end{aligned}$$

- If HFI is operating at 2,000 units or more, then machine B should be chosen, and if it is operating at below 2,000 units, then machine A should be chosen.
- For example, at 3,000 units, the total cost of machine A is $5,000 + ($10 * 3,000) = $35,000, while the cost of machine B is $15,000 + ($5 * 3,000) = $30,000, and thus the advantage goes to machine B at this sales level.

Sales Commissions and Salaries Example Three

- Another cost planning use of CVP analysis is to determine the most cost-effective means to manage selling costs.

- To illustrate, HFI management is reviewing sales salaries and commissions and finds that $1,000 of the monthly $5,000 fixed costs is for sales salaries, and that $7.50 of the $35.00 unit variable cost is a 10 percent sales commission. Suppose HFI management is considering a $450 increase in salary with an expected reduction in commission rate. How much must management reduce the commission rate to keep profits the same, assuming that sales volume and all other factors remain unchanged?
- With the proposed changes in variable and fixed costs to accommodate the new salary and commission plan, fixed costs increase by $450 per month and variable costs decrease as a result of the decrease in the commission rate, *r:*

$$v = \text{Commision rate} \times \text{Sales price} + \text{Other noncommission-based unit variable costs}$$

$$v = (r \times \$75) + \$27.50$$

And: F = Current monthly fixed costs + Increase in monthly salary

$F = \$5,000 + \$450 = \$5,450$ per month, or $\$65,400$ per year

Now we use the CVP model to solve for v:

$$Q = \frac{F + N}{p - v}$$

$$v = p - \frac{F + N}{Q}$$

and substituting for v and F with $N = \$48,000$ and $Q = 2,700$ as before:

$$(r \times \$75) + \$27.50 = \$75 - [(\$65,400 + \$48,000)/2,700]$$

$$r = 0.0733$$

CVP Analysis with Two or More Products

- Suppose now that the company has several, perhaps hundreds or thousands, of products.
- It would be difficult to calculate and to interpret that many breakeven points.
- Moreover, it would be difficult to determine how the firm's fixed costs should be allocated among these products.
- For this reason, a common approach for firms with many products is to use the contribution margin ratio (CMR) approach explained earlier.
- This approach allows the firm to estimate breakeven in dollar sales, based on an estimate of the weighted-average CMR for all of its products.

CVP Analysis with Two or More Products

- For example, if the firm has a policy of setting price for all its products at twice variable cost, then CMR is estimated as follows, where $p = 2v$, or $v = \frac{1}{2}p$:

$$\text{CMR} = (p - v)/p = \left[p - \tfrac{1}{2}p\right]/p = 50\%$$

Using the CMR approach, a firm with $100 million in fixed costs and a CMR of 50 percent would have a breakeven of $200 million sales:

$$Y = \$100/.5 = \$200,000,000$$

- To illustrate, we use the example of Windbreakers, Inc., which sells light-weight sport/ recreational jackets. Windbreakers has three products, Calm, Windy, and Gale. Relevant information for these products is in Exhibit 9.9 . The total fixed costs for the period are expected to be $168,000, and we assume that Windbreakers' sales will remain constant in sales dollars, at 50 percent, 40 percent, and 10 percent, respectively, for the three products.
- From this information, we can calculate the weighted-average CMR as follows:

$$0.5(0.2) + 0.4(0.25) + 0.1(0.1) = 0.21$$

	Calm	Windy	Gale	Total
Last period's sales	$750,000	$600,000	$150,000	$1,500,000
Percent of sales	50%	40%	10%	100%
Price	$ 30	$ 32	$ 40	
Unit variable cost	24	24	36	
Contribution margin	$ 6	$ 8	$ 4	
Contribution margin ratio	0.20	0.25	0.10	

The breakeven point in dollars for all three products can be calculated as follows:

$$Y = \$168,000/0.21 = \$800,000$$

This means that for Windbreakers to break even, $800,000 of all three products must be sold in the same proportion as last year's sales mix. The sales for each product are as follows:

For Calm	0.5($800,000) = $400,000	(13,334 jackets at $30)
For Windy	0.4($800,000) = 320,000	(10,000 jackets at $32)
For Gale	0.1($800,000) = 80,000	(2,000 jackets at $40)
Total	$800,000	

The sale of jackets in the correct sales mix produces exactly the breakeven contribution margin of $168,000:

$$\$6(13,334) + \$8(10,000) + \$4(2,000) = \$168,000$$

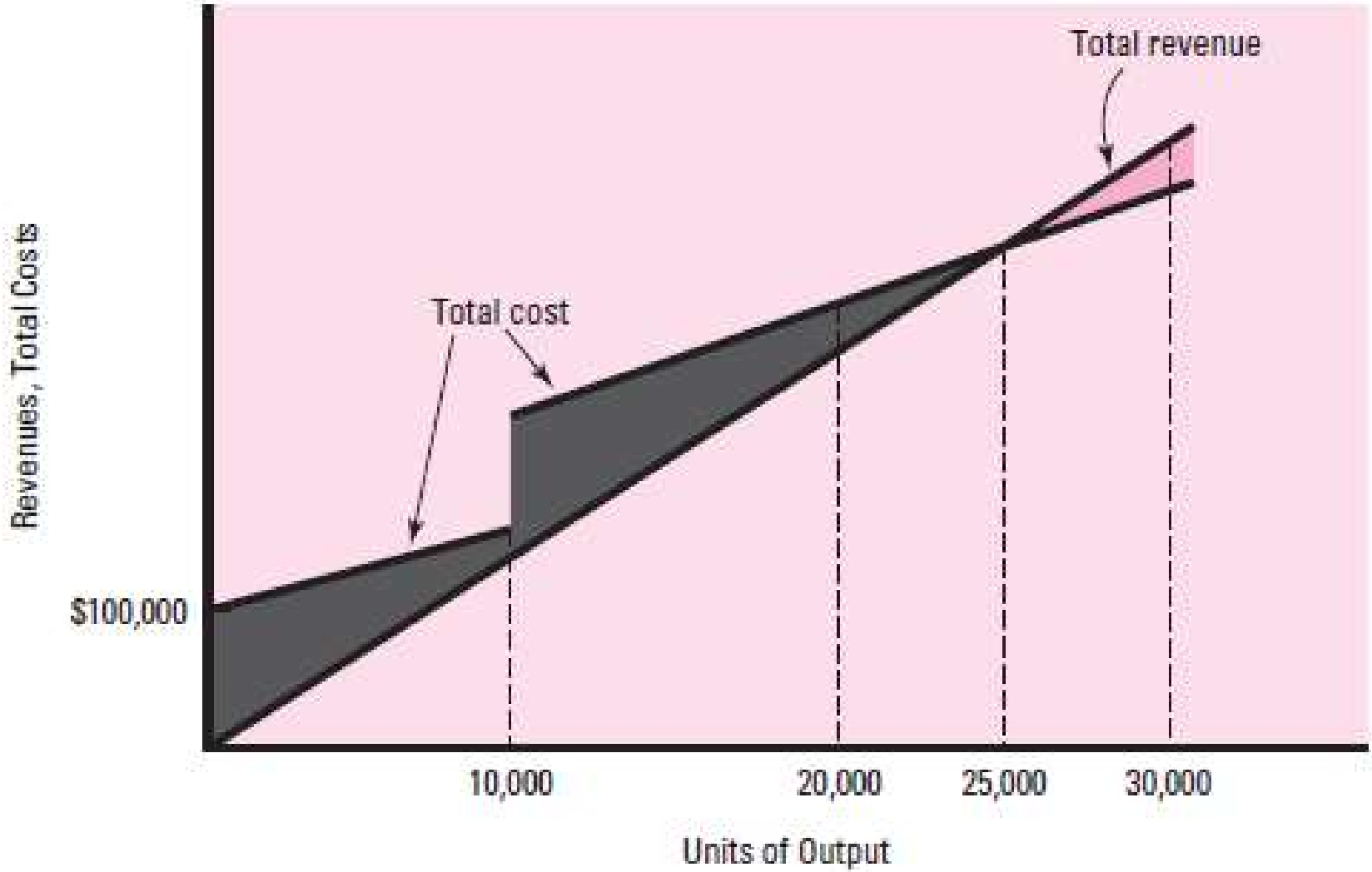
Assumptions and Limitation of CVP Analysis

- **Linearity, the Relevant Range, and Step Costs**
  - The CVP model assumes that revenues and total costs are linear over the relevant range of activity.
  - Although actual cost behavior is not linear.
  - So that within a given limited range of output, total costs are expected to increase at an approximately linear rate.
  - The caution for the manager is therefore to remember that the calculations performed within the context of a given CVP model should not be used outside the relevant range.

Assumptions and Limitation of CVP Analysis

- ***Step Costs***
- As illustrated in Exhibit 9.10 , the cost behavior under examination may be so “lumpy” (step costs) that an approximation via a relevant range is unworkable. Although CVP analysis can be done, it becomes somewhat more cumbersome. Exhibit 9.10 illustrates a situation with a price of $18, a unit variable cost of $10, an initial fixed cost of $100,000, and an incremental fixed cost of another $100,000 when output exceeds 10,000 units. The expenditure of the additional fixed cost provides capacity for up to 30,000 units. A CVP analysis requires that the manager determine the breakeven point for each range (below and above the point at 10,000 units).

$$Q = f/(p - v) \quad \$200{,}000/(\$18 - \$10) = 25{,}000 \text{ units}$$


Total revenue
Total cost
Revenues, Total Costs
$100,000
10,000
20,000
25,000
30,000
Units of Output

Identifying Fixed and Variable Costs for CVP Analysis

- ***Fixed Costs***
  - In a short-term analysis, relevant fixed costs are those expected to change with the introduction of the new product.
  - These include costs of any new production facilities, salaries of new production personnel, and similar costs.
  - If a new product does not require any new fixed costs because existing facilities and personnel can handle the added production, what is the breakeven point?
  - For a *short-term analysis,* the breakeven point is zero since the new product must cover no new fixed costs.
  - That is, each product sold, beginning with the first, contributes to profit in the excess of price over variable cost.
  - In contrast, for a *long-term analysis* of breakeven, all current and expected future fixed costs associated with the production, distribution, and sale of the product are relevant.
  - The long-term view has been taken.

Identifying Fixed and Variable Costs for CVP Analysis

- ***Variable Costs***
  - In measuring variable cost, the management accountant must be careful to include all relevant variable costs, not only production costs but also selling and distribution costs.
  - Thus, expense for commissions is properly included as a unit variable cost.
  - Any transportation or warehousing costs, if they change with level of output, are relevant.